

A Time-Homogeneous, SABR-Consistent Extension of the LMM: Calibration and Numerical Results

Riccardo Rebonato
Royal Bank of Scotland
Oxford University (OCIAM)
Imperial College, Tanaka Business School, London

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Abstract

We propose an extension of the LMM that recovers the SABR caplet prices almost exactly for all strikes and maturities. The dynamics of the volatility is chosen so as to be consistent across expiries, to be financially motivated and to make the evolution of the implied volatilities as time homogeneous as possible. Given the SABR parameters, the associated LMM parameters are found with minimal numerical work.

1 Motivation

The SABR and the LIBOR Market Model (LMM) have become industry standards for the pricing of plain-vanilla and complex interest-rate products, respectively.¹ (For a description of the SABR model see, eg, Hagan et al (2002). The LMM is described, eg, in Brace, Gatarek and Musiela (1996) and Jamshidian (1997)). While similar, the two models do not directly ‘talk to each other’. Ultimately, the SABR approach provides a series of independent ‘snapshots’ of the caplet smiles (one snapshot for each maturity), but does not link these still frames into a coherent ‘movie’ (ie, does not prescribe a well-defined joint dynamics). For the limited fitting and interpolation purposes of the SABR approach, this is perfectly correct, because it deals with the pricing of European options, for which, roughly speaking, it is only the terminal variance (in a diffusive setting, the realized quadratic variation) that matters.

Several stochastic-volatility extensions of the LMM exist that do provide a consistent dynamic description of the evolution of the forward rates (see, eg, Andersen and Andreasen (1998), Joshi and Rebonato (2003), Rebonato

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and Joshi (2004), Rebonato and Kainth (2004)), but these extensions are not equivalent to the SABR model. Since, for better or for worse, SABR has become the market standard to describe the caplet smile surface, it follows that the current stochastic-volatility extensions of the LMM fail to recover the observed market prices of caplets for arbitrary strikes. In other words, the middle ‘M’ in LMM is becoming more and more questionable.

This paper attempts to provide a simple, financially-justifiable, easy-to-calibrate, computationally-affordable and SABR-compatible extension of the LMM. More precisely, we would like to specify a financially-motivated dynamics for the LMM forward rates and volatilities that very closely matches the SABR prices. This can, of course, be done in several ways. We set ourselves, however, stringent desiderata in terms of computational ease of calibration and financial justifiability of the approach. These are outlined below.

2 Desiderata

In the deterministic-volatility (no-smile) LMM, it is well known (see, eg, Brigo and Mercurio (1991) or Rebonato (2002)) that the desirable feature of time homogeneity of the caplet implied volatility surface can only be achieved if the instantaneous volatilities of the forward rates, $\sigma_i(t, T_i)$, are purely a function, say, $g()$, of the residual time to maturity, $T_i - t$, of the associated forward rate:

$$\sigma_i(t, T_i) = g(T_i - t) \tag{1}$$

A common choice for $g(T_i - t)$ (see, eg, Brigo and Mercurio (2001), Rebonato (2002, 2005), White and Rebonato (2007) for a justification and description of the properties of this function) is:

$$\sigma_i(\tau) = (a + b\tau) \exp(-c\tau) + d \tag{2}$$

with $\tau = T - t$. This is the functional form used in the following, but the treatment below does not depend on the specific form chosen.

For practical applications one wants to give a simple (and therefore, hopefully, analytically integrable) form to the function $g()$. When this is done however, perfect recovery of an exogenous set of at-the-money prices cannot in general be recovered by such a function even in the absence of smiles. A common approach to overcome this problem is to ‘correct’ the time homogeneous function as little as possible by introducing expiry-specific factors (k_i) chosen so as to ensure that

$$\widehat{\sigma}_i T_i^2 = k_i^2 \int_0^{T_i} g(T_i - t)^2 dt \tag{3}$$

where $\widehat{\sigma}_i$ indicates the ‘implied’ (root-mean-squared) volatility of the i th caplet. If the function $g()$ is parametrized by a set of coefficients $\{\alpha_k, k = 1, 2, \dots, m\}$, then a good fitting procedure is to impose that the m parameters α should be chosen so as to ensure that the correction factors k_i are as close to unity for all forward rates. When this constraint is imposed, it is generally found

that instantaneous-volatility functions for forward rates naturally display the humped shape that has become part of market lore. See Dodds (1998) for an empirical study, and Rebonato (2005) and White and Rebonato (2007) for indirect evidence from swaption prices.

We would like to retain this intuition in our extension of the LMM. Unfortunately, a trivial ‘transliteration’ of the SABR approach into the LMM (whereby a SABR-like specification of the volatility is grafted on top of the deterministic-volatility LMM) suffers from lack of time homogeneity and gives rise to ‘non-physical’ consequences. An example of this is shown in Figure 1, which presents the SABR volatility of volatility and clearly show that they are forward-rate specific. If this constant parameter were simply used in the LMM extension, different forward rates will behave differently throughout their lives even when they have the same residual time to expiry.

Fig 1 approximately here

Fig 1: The SABR volatility of volatility as a function of the caplet expiry (in years) for the fitting on 26-Sept-2006

We can try to overcome this problem in at least two simple ways. The first is by assuming that in the LMM all forward rates should have the same volatility of volatility, but, at the same time, by giving to the stochastic volatility a mean-reverting behaviour. This approach assumes that the ‘true’ volatility of volatility of the forward rates is the same constant for all forward rates, but that different values are obtained in the SABR fitting because, for its fitting purposes, it does not need to specify a mean-reverting behaviour for the volatility. In other words, in this interpretation, the SABR model is dynamically mis-specified, the mean-reversion is ‘latent’ in its parametrization, and appears indirectly as a volatility of volatility that decays with maturity.

The second route is to assume that the volatility of volatility of the various forward-rate processes should be a function of their residual time to maturity. Looked at this way, the variation across maturities of the SABR volatility of volatilities comes about because the fitted values are different root-mean-squared volatilities of the same time-homogeneous function - the different values, of course, would arise because of the different integration times for the various forward rates.

In order to specify a financially better-motivated process for the stochastic volatility, we therefore explore two avenues: either a mean-reverting (Ornstein-Uhlenbeck) dynamics for the correction factors k_i ; or a simple function of residual maturity for the volatility of volatility. In either case we try to choose the parameters *in such a way to recover some important properties of the SABR volatility process*. We try to do so by retaining the ability for the (expectation of the) volatility function to display a humped shape as a function of residual maturity. Since (see, eg, Rebonato (2002), (2005)) the pricing of complex instruments depends on covariance elements C_{ij}^k of the type

$$C_{ij}^k = \int_{t_k}^{t_{k+1}} \sigma(T_i - s) \sigma(T_i - s) \rho_{ij}^s ds \quad (4)$$

(where $[t_k, t_{k+1}]$ is the interval between the k th and the $(k+1)$ th price-sensitive event) this feature is recognized to be important for pricing. In particular, see White and Rebonato (2007) for the impact of the shape of the volatility function on swaption prices.

We hope that, by following either of the two routes above, the processes of a given forward rate and of its associated volatility should now depend on the forward rate itself mainly via the one financially significant variable, ie, its residual time to expiry. We call this desirable feature the time-homogeneity of our extension.

Finally, and most importantly, for the procedure to be effective and practical, we propose a formulation that recovers the SABR prices to a good degree of accuracy with a minimal amount of numerical burden.

3 The Method

Consider the following SABR dynamics under the terminal measure, Q^T , of forward rate f_t^T :

$$df_t^T = \sigma_t^T (f_t^T)^{\beta_{SABR}} dz_t^T \quad (5)$$

$$\frac{d\sigma_t^T}{\sigma_t^T} = v^T dw_t^T \quad (6)$$

$$E [dz_t^T dw_t^T] = \rho dt \quad (7)$$

The SABR dynamics is therefore fully described by the initial conditions, f_0^T, σ_0^T , and by the expiry-dependent parameters β_{SABR}, ρ, v^T .²

We now move to the LMM specification. We work under the same terminal measure, Q^T , under which the forward rate is driftless. The choice of a different measure will simply introduce the familiar drift correction terms (see, eg, Jamshidian (1997) or Brace, Gatarek and Musiela (1996)). Since these drift correction terms are purely a function of the volatilities (and correlations), once the LMM volatility process is specified the no-arbitrage corrections are uniquely determined.

The SABR parameters above are assumed to be available from a previous SABR fitting for all maturities. They implicitly determine the caplet prices for all strikes, and for all the maturities for which they have been fitted. We want to determine the parameters of an LMM model such that the LMM caplet prices for all the same strikes and maturities are as close as possible to the SABR caplet prices.

To this effect consider the dynamics of the forward rate of maturity T under the same measure, Q^T , in the LMM:

$$df_t^T = s_t^T (f_t^T)^{\beta_{LMM}} dz_t^T \quad (8)$$

²To lighten notation, we omit the dependence of the correlation ρ and of the exponent β_{SABR} on the expiry T .

$$s_t^T = k_t^T g_t^T \quad (9)$$

$$dp(t, T) = dk_t^T \quad (10)$$

with $g_t^T = g(T - t)$ and

$$p(t, T) = \frac{s_t^T}{g_t^T} \quad (11)$$

We can interpret the quantity $p(t)$ as the ratio of the total instantaneous volatility of the T -expiry forward rate to the deterministic and time-homogeneous component of the same volatility. Note that, if k_t^T were a deterministic function of t and T (or a constant), then we would be in the traditional deterministic-volatility LMM setting. Instead, we prescribe for k_t^T a process of the type (see, eg, Schwartz (1997):

$$dk_t^T = k_t^T a^T (RVL - \ln k_t^T) dt + k_t^T h_t^T dw_t^T + k_t^T \epsilon dt \quad (12)$$

$$E[dw_t^T dz_t^T] = r dt \quad (13)$$

This formulation will allow us to explore both the mean-reverting and the time-homogeneous volatility of volatility case. If the first avenue is taken, in keeping with the intuition that applies to the deterministic-volatility case, we would like k_t^T to revert to 1. By so doing, the volatility function would be ‘attracted’ over time to the most time-homogeneous solution compatible with the current observed prices. Hence we posit

$$RVL = 0 \quad (14)$$

and

$$dp_t^T = k_t^T a^T (RVL - \ln k_t^T) dt + k_t^T h_0 dw_t^T + k_t^T \epsilon dt \quad (15)$$

$$p(0) = \frac{s(0)}{g(T - 0)} \quad (16)$$

Define now $y(t) = \ln k(t)$. Applying Ito’s lemma (and dropping superscripts for ease of notation) one obtains

$$dy_t = a(m - y_t) dt + h_0 dw_t + \epsilon dt \quad (17)$$

with

$$m = -\frac{h_0^2}{2a} \quad (18)$$

A solution to the SDE can be found to be

$$y(t) = \epsilon t + m + \exp(-at) [y(0) - m] + \int_0^t h_0 \exp[a(t - s)] dw_s \quad (19)$$

Therefore, for $t > 0$,

$$s_t^T = g_t^T * \exp \left[\epsilon t + m + \exp(-at) [y(0) - m] + \int_0^t h_0 \exp[a(t - s)] dw_s \right] \quad (20)$$

This fully specifies the mean-reverting extension of the LMM we intend to work with.

As for the second route, we simply set $a = 0$ in the expressions above, but impose that the volatility of volatility should have the functional form

$$h(t, T) = h(T - t) \quad (21)$$

with the function $h(\cdot)$ parametrized by a set of parameters $[l, m, n, \dots]$. Therefore

$$k_t^T = k_0^T \exp \left[\int_0^T -\frac{1}{2} h^2(T - s) ds + h(T - s) dw_s \right] \quad (22)$$

and

$$s_t^T = g_t^T k_0^T \exp \left[\int_0^T -\frac{1}{2} h^2(T - s) ds + h(T - s) dw_s \right] \quad (23)$$

The problem is how to choose the parameters of the two specifications in a way compatible with the desiderata outlined above. We note in passing that in a deterministic-volatility setting the quantity is fully determined by the requirement that each caplet should be perfectly priced. See Equation (3). This is no longer the case in the stochastic-volatility setting.

4 Calibration

To start, the following obvious choices can be made:

$$r = \rho \quad (24)$$

$$\beta_{SBR} = \beta_{LMM} \quad (25)$$

Having made these easy choices, the next question is whether a time-homogeneous description of the joint dynamics of the forward rates and their volatilities should be attempted by using a mean-reverting process for the volatility or a maturity-dependent volatility of volatility. Let's examine the first possibility first.

If the market 'believed' in a mean-reverting process for the volatility, we would expect the root-mean-squared volatility of volatility to be a monotonically declining function of maturity. This indeed happens to be the case on many fitting days, but Fig 1 shows a trading day when, because of the hump in the volatility-of-volatility-as-a-function-of-maturity graph, a mean-reverting process with constant parameters could not account for the SABR-fitted market values. Trading days with a humped volatility-of-volatility curve are more rare, but by no means exceptional. This already creates a serious problem. There are more fundamental reasons, however, to reject the mean-reverting solution. As we show below, obtaining a very good fit to the market prices across strikes *for a fixed maturity* is indeed possible with a mean-reverting process. However, the same parameters do not provide an acceptable fit for different trading days. This is particularly true when the volatility of volatility function is humped, but

is a general finding. A variation of the mean-reverting parameters with forward-rate expiry goes against the grain of what we attempt to achieve. We therefore abandon this route.

We move to the second possibility, ie, we assume a dependence of the volatility of volatility on the residual time to maturity of the underlying forward rate, $h_t^T = h(T - t)$. We choose a function of the form:

$$h_\tau = (\alpha + \beta\tau) \exp(-\gamma\tau) + \delta \quad (26)$$

with $\tau = T - t$. The outstanding question is how to choose the parameters α, β, γ and δ in Equation (26) and a, b, c and d in Equation (2). We proceed as follows.

The formal solution for the SABR evolution to time t of the T -maturity forward rate is

$$\begin{aligned} f_t^T &= f_0^T + \int_0^T (f_s^T)^\beta \sigma_s^T ds = \\ &= f_0^T + \int_0^T (f_s^T)^\beta \sigma_0^T \exp\left[\int_0^s -\frac{1}{2}v^2 dt + vdw_u\right] ds \end{aligned} \quad (27)$$

The equivalent expression for the LMM evolution (with zero reversion speed) is:

$$\begin{aligned} f_t^T &= f_0^T + \int_0^T (f_s^T)^\beta s_s^T ds = \\ &= f_0^T + \int_0^T (f_s^T)^\beta k_s^T g_s^T ds \end{aligned} \quad (28)$$

with

$$k_s^T = k_0^T \int_0^s \exp\left[-\frac{1}{2}h^2(T-u)du + h(T-u)dz_u\right] \quad (29)$$

In a deterministic-volatility setting one would choose the parameters a, b, c and d in such a way as to match as closely as possible not the root-mean-squared volatility $\hat{\sigma}_i$. See Equation (3). In a stochastic-volatility setting there is no single root-mean-squared volatility, as it depends on the realized path. In this more complex setting, it is plausible to choose the parameters a, b, c and d in such a way as to match as closely as possible the expectation at time 0 of σ_s^T , σ_0^T . More precisely, we minimize over a, b, c and d the sum, χ^2 , of the squared discrepancies:

$$\chi^2 = \sum_i^N (\sigma_0^{T_i})^2 T_i - \int_0^{T_i} [(a + b\tau_i) \exp(-c\tau_i) + d]^2 \quad (30)$$

where the sum over i runs over the N caplet expiries. The initial values $k_0^{T_i}$ in Equation (3) are then chosen so as to provide exact recovery of the quantities $(\sigma_0^{T_i})^2 T_i$. To the extent that the chosen function $g(\tau)$ allows for a good fit to the dependence of the initial SABR value $\sigma_0^{T_i}$ on the maturities T_i , these correction

factors will all be close to 1. Empirical fits, discussed in the following, show that this is indeed the case.

Rebonato (2002) shows that, as long as the quantity $(\sigma_0^{T_i})^2 T_i$ is a monotonically increasing function of T_i , it is always possible to find a time-homogeneous function $g(\tau)$ such that the recovery of the quantities $(\sigma_0^{T_i})^2 T_i$ is exact. In the present study we prefer to use a small number of parameters and a simple (and integrable) functional form, and to pay the price of a small degree of discrepancy.

Using the fitted SABR values, $v(T)$, we then follow a very similar procedure, ie, we determine the coefficients α, β, γ and δ that minimize the sum, χ^2 , of the squared discrepancies:

$$\chi^2 = \sum_i^2 (v^T)^2 T_i - \int_0^{T_i} [(\alpha + \beta\tau_i) \exp(-\gamma\tau_i) + \delta]^2 \quad (31)$$

Again, small correction factors, ξ_i , then ensure the exact recovery of the root-mean-squared volatility of volatility. Fig displays the results for two such typical fits.

Fig 2 approximately here.

Fig 2: The market and fitted volatility of volatility function against caplet expiry (in years) for 26-09-2006

The main justification for this calibration lies in its simplicity. For instance, rather than matching the square of the expectation at time 0 of σ_s^T , $(\sigma_0^{T_i})^2$, it would be more appealing to match the expectation at time 0 of $\int_0^T (\sigma_s^T)^2 ds$. The resulting expressions, however, become more cumbersome, and the numerical search less straightforward. In either case, there is no rigorous justification for doing so, as, clearly, the expected price is not equal to the price with the expected variance. The success of the procedure will depend on the degree of ‘convexity’ that is neglected in the approximation³. This is an empirical issue that is dealt with in the next section.

5 Results

We have attempted the recovery of the SABR prices with both approaches. We present the results with zero reversion speed and a time-homogeneous volatility of volatility first.

³By matching the expectation at time 0 of σ_s^T rather than the expectation of $\int_0^T (\sigma_s^T)^2 ds$, we are neglecting two convexities (two Jensen inequality effects): one coming from the fact that the square of the expectation is not equal to the expectation of the square; the other stemming from the difference of the expectation over prices from the price calculated with the expected variance.

As can be seen from Figures (3) to (6), after the calibration outlined above is carried out and typical market values are obtained, the agreement between the SABR and the LMM prices ranges from good to excellent over a very large range of strikes and maturities. (The parameters used were obtained from a market fit). We stress that the same parameters α, β, γ and δ and a, b, c and d are used for all the strikes and all the maturities for a given trading day.

Fig 3: SABR and LMM implied volatilities ($\beta_{SABR}=0.75$, $T= 4$ years, $v = 27.5\%$, $\rho=-0.1$, $f(0) = 4.00\%$)

Fig 4: SABR and LMM implied volatilities ($\beta_{SABR}=0.75$, $T= 10$ years, $v = 25\%$, $\rho=-0.1$, $f(0) = 4.00\%$)

Fig 5: SABR and LMM implied volatilities ($\beta_{SABR}=0.75$, $T= 20$ years, $v = 20\%$, $\rho=-0.1$, $f(0) = 4.00\%$)

Fig 6: SABR and LMM implied volatilities ($\beta_{SABR}=0.75$, $T= 30$ years, $v = 20\%$, $\rho=-0.1$, $f(0) = 4.00\%$)

For the procedure suggested in the previous section to make sense the correction factors, ξ_i , must turn out to be all close to 1. Fig 7 shows that this is indeed the case for a typical trading day (26-09-06).

Fig 7: The correction factors as a function of caplet expiry (in years).

The pricing discrepancies are reported in Figs (8) to (11) for three maturities. For all maturities the nature of the error appears to be approximately quadratic as the distance from the at-the-money strike increases. This convexity effect increases as a function of expiry, but, for typical market-plausible values of the SABR parameters, the error in implied volatility remains below one (yield) vega even for maturities of twenty or thirty years.

Fig 8: The errors as a function of the option strike for the 4-year expiry reported in Fig 3.

Fig 9: The errors as a function of the option strike for the 10-year expiry reported in Fig 4.

Fig 10: The errors as a function of the option strike for the 20-year expiry reported in Fig 5.

Fig 11: The errors as a function of the option strike for the 30-year expiry reported in Fig 6.

Generally, one can say that the quality of the approximation deteriorates as the quantity σ_0^T increases, or as the maturity increases. However we observe from market fits that σ_0^T tends to decline for increasing maturities, thereby producing a compensating effect. We stress, however, that these results have been obtained with recent market-typical parameters (US\$). For all currencies, markets have been experiencing in the last few years a period of low interest-rate volatilities. While we have checked the robustness of the results to reasonable variations around the market-fitted SABR parameters, it cannot be guaranteed

that the approximations presented here would still work under different regimes (higher volatilities, and higher volatility of volatility).

Since fast analytic approximations are available for the SABR prices, these constitute a simple and ideal contravariate variable to reduce the error further. It should be kept in mind that the asymptotic expansions typically used (Hagan et al (2002)) are themselves approximation, and that these begin to break down for out-of-the-money strikes and long maturities. It is debatable if currently market prices are quoted on the basis of the ‘true’ SABR process or of the asymptotic approximation that has been universally adopted. In order not to confuse sources of error coming from different approximations we have therefore not included the contravariate correction, and we present the comparison between the LMM extension and the ‘true’ SABR process.

Finally we show in Fig 12 that it is indeed possible to obtain a very good fit for a *fixed maturity* also using a mean-reverting process and a constant volatility.⁴ As mentioned above, however, not only is the calibration much more time-consuming, but the constant parameters that describe well one expiry do not capture the prices of the caplets of different expiries on the same trading day.

Fig 12: Matching the square integrated variance with a mean-reversion process and constant volatility; $T = 5$ yr, 50% $v=50\%$ $\rho = -0.3$, $\beta_{SABR} = 1$.

6 Conclusions and Extensions

We have presented a simple-to-calibrate and financially-motivated extension of the LMM that recovers to a high degree of accuracy the caplet prices produced by the SABR model. Since, in turn, this has become the market standard for plain-vanilla caplet prices, the proposed procedure extends the LMM in such a way that a small number of time-homogenous parameters simultaneously recover the market caplet prices for all strikes and maturities. The calibration burden on top of what is required for a no-smile LMM is minimal. The computational burden of Monte Carlo applications can be reduced by using an efficient contravariate scheme.

Further substantial numerical improvements can be obtained if a displaced diffusion (DD) process is posited for the forward rates, instead of a CEV process. This can be done efficiently and accurately thanks to the correspondence between CEV and DD processes first highlighted by Marris (1998), and then studied further by Svoboda (2006). We have not reported the results of these tests because they would cloud the discussion of the approximations suggested in this work. For Monte Carlo applications, however, the prior transformation of the CEV process for the forward to the ‘equivalent’ DD prices is strongly recommended.

⁴The computations carried out by Dr Matthew Dodson and Dr Dherminder Kianth are gratefully acknowledged.

One more aspect lends itself to further interesting work. The current implementation is clearly valid for any correlation matrix, $\widehat{\rho}_{ij}$, among the increments of the the volatilities of forward rates different maturities:

$$E \left[dw_t^{T_i} dw_t^{T_j} \right] = \widehat{\rho}_{ij} dt \quad (32)$$

The simplest choice is a ‘‘one-factor model’’:

$$E \left[dw_t^{T_i} dw_t^{T_j} \right] = 1 dt \quad \text{for any } i, j \quad (33)$$

However, volatilities (or, at least, implied ones) are not observed to move in lockstep. See, eg, Rebonato and Joshi (2004) for a study of the principal components of the swaption matrix. Information about a richer correlation structure could therefore be gleaned along two distinct routes:

- an empirical approach, where the statistical properties of the volatility of implied volatilities (seen as integrals of instantaneous volatilities) are analyzed;
- an ‘implied’ approach, where information from the smile structure of swaptions is brought into play.

We leave these topics for further research.

Finally we point out that, after the SABR fitting is carried out, also the correlation coefficients ρ and the exponent β_{SABR} typically display a variation as a function of expiry. As a consequence, the calibration choices (24) and (24) do not provide a truly time-homogeneous solution: different forward rates remain ‘different’ even when the residual time to maturity is the the same. The same approach suggested for the volatility of volatility could of course be followed. However, it is well known that the parameters ρ and β_{SABR} are strongly negatively correlated: both mainly influence the slope of the smirk (more negative correlation and exponents closer to zero make the smile steeper), and the same quality of fit can often be obtained with very different combinations of ρ and β_{SABR} . For this reason, establishing a dependence on the forward-rate expiry of either paramter is difficult. Indeed, in order to avoid instability of the fits, many trading houses fix either paramter to a given value, and only optimize over the other. When this is done, a modest variation across expiries is found in the free parameter. As an example, we show in Fig (13) the dependence on expiry of the correlation coefficient obtained after market fitting for a fixed exponent of $\beta_{SABR} = 0.75$. For these reasons, and to limit the number of fitting parameters, we have simply accepted the calibration choices (24) and (24).

Fig 13: Variation of the SABR correlation coefficient, ρ , as a function of expiry (27-Jan-2007)

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